NORTH SYDNEY BOYS HIGH SCHOOL

2009 YEAR 12 HSC
ASSESSMENT TASK 1

Mathematics

General Instructions

- Working time – 65 minutes
- Write on one side of the paper (with lines) in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**

Total Marks (59)

- Attempt all questions

Class Teacher:
(Please tick or highlight)

- O Mr Barrett
- O Mr Fletcher
- O Mr Ireland
- O Mr Lowe
- O Mr Rezcallah
- O Mr Trenwith
- O Mr Weiss

Student Number: ....

Question	1	2	3	4	5	6	Total	Total
Mark	/8	/9	/10	/10	/12	/10	/59	/100

2009 - Past paper.

Start each question on a new page.

Question 1 (8 marks) Start a new page. **Marks**

(a) Differentiate with respect to x:

(i) $(x^2 + 3x)^4$ **2**

(ii) $\frac{x^2+1}{x-2}$ **2**

(b) (i) Find the x coordinate of the point on the graph of $y = 5 - 14x - 2x^2$ where the tangent is parallel to the line $y = -2x + 7$ **2**

(ii) Hence find the equation of this tangent. **2**

Question 2 (9 marks) Start a new page.

(a) In a certain arithmetic series, the second term is 19 and the eighth term is 37.

(i) Show that the common difference is 3 **2**

(ii) Find the value of the 51st term. **2**

(b) Evaluate $\sum_{k=2}^{5} (k-1)^2$ **2**

(c) By considering the recurring decimal $0 \cdot 4\dot{5}$ as the **sum of an infinite geometric series**, express $0 \cdot 4\dot{5}$ in the form $\frac{a}{b}$. **3**

Question 3 (10 marks) Start a new page.

Consider the curve given by $y = 1 + 3x - x^3$, for $-2 \leq x \leq 3$.

(a) Find the coordinates of the stationary points and determine their nature. **4**

(b) Find the coordinates of any points of inflexion. **2**

(c) Sketch the curve in the domain $-2 \leq x \leq 3$. **3**

(d) What is the minimum value of the function in the domain $-2 \leq x \leq 3$? **1**

Start each question on a new page.

Question 4 (10 marks) Start a new page.

(a) The current i in a certain resistor as a function of the power P developed in the resistor is given by $i = 2.6\sqrt{P}$.
Find the rate of change of i with respect to P when $P = 4$. **2**

(b) A particle moves in a straight line and after t seconds its velocity v metres per second given by $v = 12t - 3t^2$.

(i) When is the particle at rest? **2**

(ii) What is the acceleration at $t = 2$? **2**

(c) Find all values of x for which the function $f(x) = 8x^2 - 24x + 5$ is increasing. **2**

(d) A curve $y = f(x)$ has the following properties:

$f(x) > 0;\ \ f'(x) > 0;\ \ f''(x) < 0.$

Sketch a curve satisfying these conditions. **2**

Question 5 (12 marks) Start a new page.

(a) The first term of a geometric series is 7 and the 6th term is 1 701.

(i) Find the common ratio. **2**

(ii) Calculate the sum of the first ten terms. **2**

(b) The sum S_n of the first n terms of a certain series is $2n + 3n^2$, for $n \geq 1$.
Find an expression for the nth term T_n of this series. **3**

(c) Mick received 30 tonnes of topsoil for his front yard. He uses a wheel barrow which can hold 150kg to spread the soil.

(i) How many loads in the wheel barrow will he use? **1**

He begins at the pile of topsoil and deposits the first load 3 metres from the pile. Each successive load is dumped half a metre further from the pile, in a straight line.

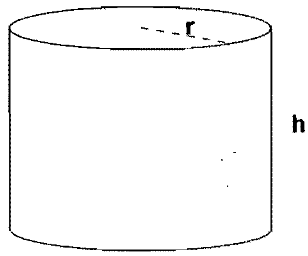
(ii) How far from the pile will he leave the final barrow load? **2**

(iii) What is the total distance that Mick will travel with the wheel barrow if he finishes back at his starting point? **2**

Start each question on a new page.

Question 6 (10 marks) Start a new page.

(a) The Green group at NSBHS wishes to build a new water tank. It is to be in the shape of a closed cylinder of radius r metres and height h metres, as shown in the diagram:



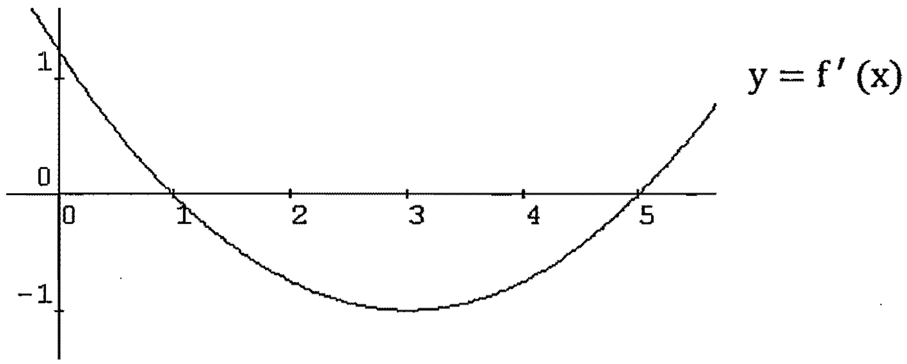
(i) The surface area of metal to be used in the tank is 30m^2.

Use that fact to show that $h = \frac{15}{\pi r} - r$ **2**

(ii) Show that the volume, $V\text{m}^3$ of the tank is given by $V = 15r - \pi r^3$ **1**

(iii) Find the radius if the volume of the tank is to be maximised. **3**

(b) The following diagram shows the graph of the gradient function of $f(x)$.
For what value of x does $f(x)$ have a local minimum?
Justify your answer. **2**



(c) By referring to the derivative of the function $f(x) = \sqrt{x-1}$, explain why the curve $y = \sqrt{x-1}$ has no turning points. **2**

2009 HSC — Assessment Task 1 Suggested solutions

Q1. (a) (i) $\frac{d}{dx}(x^2+3x)^4 = (2x+3)\cdot 4\cdot(x^2+3x)^3$ ✓

$= (8x+12)(x^2+3x)^3$ ✓

(ii) $\frac{d}{dx}\left(\frac{x^2+1}{x-2}\right) = \frac{(x-2)\cdot(2x)-(x^2+1)(1)}{(x-2)^2}$ ✓

$= \frac{x^2-4x-1}{(x-2)^2}$ ✓

(b) (i) $y = 5-14x-2x^2$

$\therefore y' = -14x-4x$

Gradient of $y=-2x+7$ is -2

$\therefore -14-4x = -2$ ✓ correctly equating

$\therefore x = -3$ ✓ correct answer

(ii) at $x=-3$, $y = 5-14(-3)-2(-3)^2$

$\therefore y = 29$ ✓ for y value

$\therefore \quad y-29 = -2(x+3)$

$\therefore y = -2x+23$ ✓ for equation

(i.e. $2x+y-23=0$)

Q2.

(a) (i) $T_2 = a + d = 19$ ------ ①

$T_8 = a + 7d = 37$ ---- ②

✓ for working

①−② →

$6d = 18$

$d = 3$

✓ for $d=3$

(ii) $T_{51} = a + 50d$

From (i), $a = 19 - d = 16$

$\therefore T_{51} = 16 + 50(3)$ ✓ correct sub. in

$\therefore T_{51} = 166$ ✓ correct answer

(b) $\sum_{k=2}^{5} (k-1)^2 = 1^2 + 2^2 + 3^2 + 4^2$ ✓ correct working

$= 1 + 4 + 9 + 16$

$= 30$ ✓ correct answer

(c) $0.4\dot{5} = 0.4 + \underbrace{0.05 + 0.005 + 0.0005 + \cdots}$

G.P. with $a = 0.05$, $r = 0.1$ ✓ correct G.P. parameters

$\therefore S_\infty = \dfrac{0.05}{1 - 0.1}$ ✓ correct sub. into S_∞

$= \dfrac{5}{90}$

$\therefore 0.4\dot{5} = \dfrac{4}{10} + \dfrac{5}{90}$

$= \dfrac{36+5}{90}$

$= \dfrac{41}{90}$ ✓ correct answer

Q3. $y = 1 + 3x - x^3 \quad , \quad -2 \le x \le 3$

(a) For stationary points, $y' = 0$

$\therefore y' = 3 - 3x^2 = 0$ ✓ Correct derivat equal to 0

$\therefore 3(1-x)(1+x) = 0$

$\therefore x = 1$ or $x = -1$ ✓ Correct x-coordinate

$y = 3 \qquad y = -1$

So stationary points are $(1,3)$ and $(-1,-1)$

$y'' = -6x$

$\begin{cases} \text{At } (1,3), \ y'' = -6 < 0 \quad \therefore (1,3) \text{ is max. turning point} \\ \text{At } (-1,-1), \ y'' = +6 > 0 \quad \therefore (-1,-1) \text{ is min. turning point} \end{cases}$ ✓ for testing both points ✓ for correct coordinates & nature

[Alternatively, may use y' to test the points]

(b) For inflexion, $y'' = 0 \quad \therefore -6x = 0$

$\therefore x = 0, \ y = 1$ ✓ for $(0,1)$

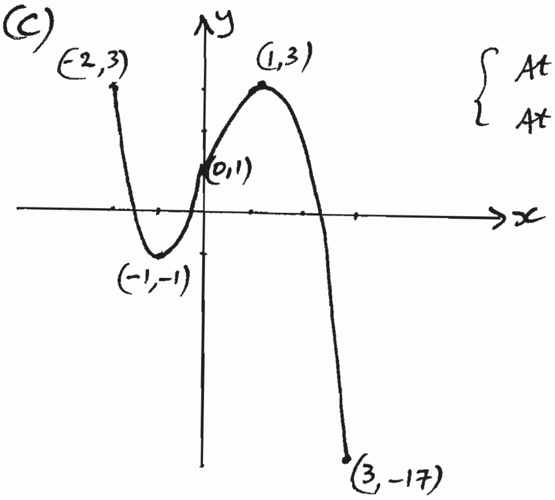
So inflexion could be $(0,1)$

Test:

x	$-0{\cdot}1$	0	$+0{\cdot}1$
y''	$+$	0	$-$

Concavity changes, $\therefore (0,1)$ is an inflexion pt. ✓ for testing

(c)



$\begin{cases} \text{At } x = -2, \ y = 1 - 6 + 8 = 3 \\ \text{At } x = 3, \ y = 1 + 9 - 27 = -17 \end{cases}$

✓ Correct shape

✓ end points

✓ for the 3 points found earlier

(d) Minimum value in domain $-2 \le x \le 3$ is -17 ✓ for the correct minimum

Q4.

(a) $i = 2.6\sqrt{P}$ $\quad \therefore \dfrac{di}{dP} = \frac{1}{2}(2.6)P^{-\frac{1}{2}}$

$$= \frac{1.3}{\sqrt{P}}$$

✓ correct derivative

$\therefore$ at $P = 4$, $\dfrac{di}{dP} = \dfrac{1.3}{\sqrt{4}} = 0.65$

✓ correct answer

(b)

(i) 'at rest' $\Rightarrow v = 0$

$\therefore \quad 12t - 3t^2 = 0$

$3t(4 - t) = 0$

$\therefore$ at rest at $t = 0$ & $t = 4$ seconds

✓ setting $v = 0$

✓ for both times

(ii) $a = \dfrac{dv}{dt} = 12 - 6t$

$\therefore$ at $t = 2$, $\quad a = 12 - 6(2)$

$\therefore a = 0$

So acceleration $= 0$ at $t = 2$

✓ correct acceleration equation

✓ correct acceleration value

(c) $f(x) = 8x^2 - 24x + 5$

$f'(x) = 16x - 24$

$f(x)$ increasing $\Rightarrow f'(x) > 0$

$\therefore \quad 16x - 24 > 0$

$x > \dfrac{24}{16}$

(ie $x > \frac{3}{2}$)

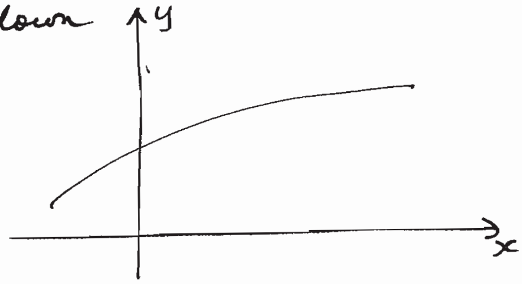
✓ sets $f' > 0$

✓ correct answer

(d) $f(x) > 0 \Rightarrow$ above x axis

$f'(x) > 0 \Rightarrow$ increasing

$f''(x) < 0 \Rightarrow$ concave down



✓✓ correct possible curve

(deduct 1 each error)

Q5.

(a) (i) $T_1 = a = 7$

$T_6 = ar^5 = 1701$

$\therefore \dfrac{ar^5}{a} = \dfrac{1701}{7}$

$r^5 = 243$

$r = 3$

✓ correct working.

✓ correct answer

(ii) $S_{10} = \dfrac{7(3^{10}-1)}{3-1}$ 9 means wrong formula — no marks

$= 206\ 668$

✓ sub. in correctly

✓ correct answer

(b) $S_n = 2n + 3n^2$

$T_n = S_n - S_{n-1}$

$= (2n+3n^2) - (2(n-1) + 3(n-1)^2)$

$= 2n + 3n^2 - (2n - 2 + 3n^2 - 6n + 3)$

$\therefore T_n = 6n - 1$

✓ sub in n-1 correctly

✓ $S_n - S_{n-1}$ if they made an attempt to sub in

✓ final answer

[alternatively, candidates may calculate successive sums $S_1, S_2, S_3 \ldots$ and discern the pattern that way.]

— If they used this method, the needed to show it was an AP algebraicly, ie. not by showing first few terms have a common difference.

(2 marks for success)

Q5 (c)

(i) number of loads $= \dfrac{30\,000}{150}$

$\therefore \quad = 200$ loads

✓ correct answer

(ii)

The distances from the pile form an A.P. with $a = 3$, $d = 0{\cdot}5$

$\therefore$ 200th load $= T_{200} = a + (n-1)d$

$= 3 + 199(0{\cdot}5)$

$= 102{\cdot}5$ m

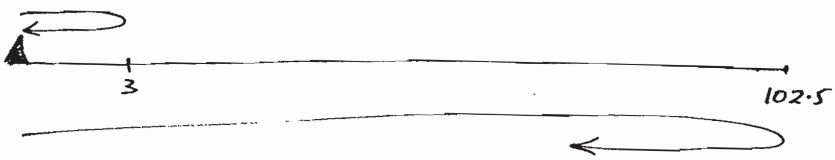
– wrong formula – no marks

– S_n – no marks

✓ correctly sub. into formula

✓ correct answer

(iii)



Total distance walked

$= 2\,(3 + 3{\cdot}5 + 4 + \dots + 102{\cdot}5)$

$= 2\left[\dfrac{200}{2}\left(3 + 102{\cdot}5\right)\right]$

$= 21{\cdot}1$ km

✓ for calculate of S_n

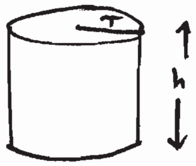
✓ for include return trips

* wrong formula – no marks

– T_n – no marks.

Q6.

(a)



(i)

Surface area $= 2\pi r^2 + 2\pi r . h$

$\therefore \quad 2\pi r^2 + 2\pi r h = 30$ ✓ correct equation

$2\pi r(r+h) = 30$

$r+h = \frac{30}{2\pi r} = \frac{15}{\pi r}$ ✓ correctly simplified

$\therefore h = \frac{15}{\pi r} - r$

(ii) $V = \pi r^2 . h$

$\therefore V = \pi r^2 \left(\frac{15}{\pi r} - r \right)$ . . . . ✓ for this calculation

$\therefore V = 15r - \pi r^3$

(iii) for max. volume, $\frac{dV}{dr} = 0$

$\therefore \quad 15 - 3\pi r^2 = 0$ . . . . ✓ correctly sets derivative $= 0$

$r^2 = \frac{5}{\pi}$

$\therefore r = \pm\sqrt{\frac{5}{\pi}}$

but $r > 0$ $\quad \therefore r = \sqrt{\frac{5}{\pi}}$ ✓ correct radius

$(\doteqdot 1{\cdot}262$ to 3 d.p.$)$

Now $\frac{d^2V}{dr^2} = -6\pi r$ ✓ for testing correctly

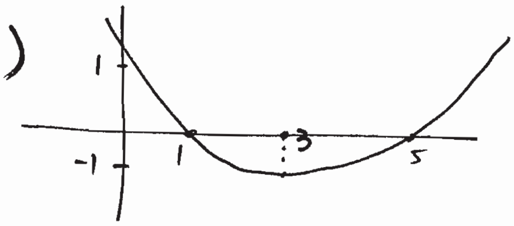
$\therefore$ at $r = \sqrt{\frac{5}{\pi}}$, $\frac{d^2V}{dr^2} < 0$

$\therefore V$ is a maximum at $r = \sqrt{\frac{5}{\pi}}$

[alternatively, test first derivative]

Q6.

(b)



Min. or max. occur when $f'(x) = 0$,
ie at $x = 1$ or $x = 5$.

For a minimum, $f'(x)$ must change from $-$ to $+$.

$\therefore$ local minimum is at $x = 5$

✓ for $x = 5$

✓ for correct reasoning

(c) $f(x) = \sqrt{x-1}$

$\therefore f'(x) = \frac{1}{2}(x-1)^{-\frac{1}{2}}$

$= \dfrac{1}{2\sqrt{x-1}}$

✓ for correct derivative

But $\dfrac{1}{2\sqrt{x-1}}$ can never equal 0

$\therefore$ there are no turning points.

✓ for correct reasoning